Practice Problems

1. Light travels through 1.00 m of water in 4.42×10^{-9} s. What is the speed of light in water?
   a. 4.42×10^{-9} m/s
   b. 4.42×10^{9} m/s
   c. 2.26×10^{8} m/s
   d. 226×10^{8} m/s

2. An astronaut on the moon receives a message from mission control on Earth. The signal is sent by a form of electromagnetic radiation and takes 1.28 s to travel the distance between Earth and the moon. What is the distance from Earth to the moon?
   a. 2.34×10^{5} km
   b. 2.34×10^{8} km
   c. 3.84×10^{5} km
   d. 3.84×10^{8} km

Check Your Understanding

3. Explain what is meant by a frame of reference.
   a. A frame of reference is a graph plotted between distance and time.
   b. A frame of reference is a graph plotted between speed and time.
   c. A frame of reference is the velocity of an object through empty space without regard to its surroundings.
   d. A frame of reference is an arbitrarily fixed point with respect to which motion of other points is measured.

4. Two people swim away from a raft that is floating downstream. One swims upstream and returns, and the other swims across the current and back. If this scenario represents the Michelson–Morley experiment, what do (i) the water, (ii) the swimmers, and (iii) the raft represent?
   a. the ether rays of light Earth
   b. rays of light the ether Earth
   c. the ether Earth rays of light
   d. Earth rays of light the ether

5. If Michelson and Morley had observed the interference pattern shift in their interferometer, what would that have indicated?
   a. The speed of light is the same in all frames of reference.
   b. The speed of light depends on the motion relative to the ether.
   c. The speed of light changes upon reflection from a surface.
   d. The speed of light in vacuum is less than 3.00×10^{8} m/s.

6. If you designate a point as being fixed and use that point to measure the motion of surrounding objects, what is the point called?
   a. An origin
   b. A frame of reference
   c. A moving frame
   d. A coordinate system

10.2 Consequences of Special Relativity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the relativistic effects seen in time dilation, length contraction, and conservation of relativistic momentum
- Explain and perform calculations involving mass-energy equivalence

Section Key Terms

binding energy	length contraction	mass defect	time dilation

proper length	relativistic	relativistic momentum
relativistic energy	relativistic factor	rest mass

Relativistic Effects on Time, Distance, and Momentum

Consideration of the measurement of elapsed time and simultaneity leads to an important relativistic effect. **Time dilation** is the phenomenon of time passing more slowly for an observer who is moving relative to another observer.

For example, suppose an astronaut measures the time it takes for light to travel from the light source, cross her ship, bounce off a mirror, and return. (See Figure 10.5.) How does the elapsed time the astronaut measures compare with the elapsed time measured for the same event by a person on the earth? Asking this question (another thought experiment) produces a profound result. We find that the elapsed time for a process depends on who is measuring it. In this case, the time measured by the astronaut is smaller than the time measured by the earth bound observer. The passage of time is different for the two observers because the distance the light travels in the astronaut's frame is smaller than in the earth bound frame. Light travels at the same speed in each frame, and so it will take longer to travel the greater distance in the earth bound frame.

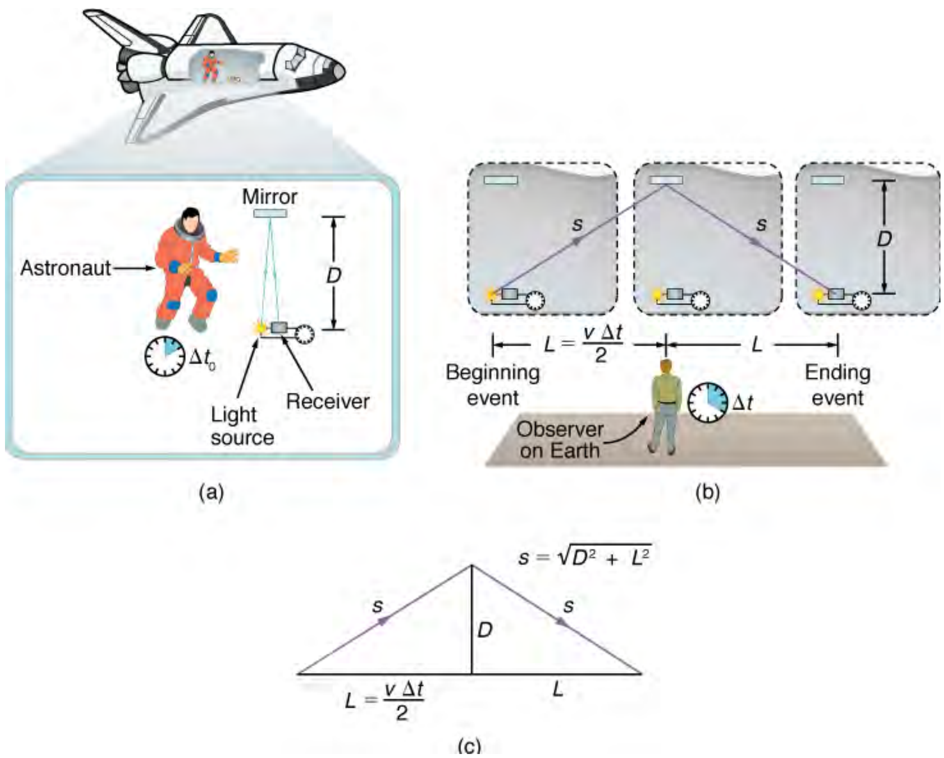

Figure 10.5 (a) An astronaut measures the time Δt_0 for light to cross her ship using an electronic timer. Light travels a distance $2D$ in the astronaut's frame. (b) A person on the earth sees the light follow the longer path $2s$ and take a longer time Δt.

The relationship between Δt and Δt_0 is given by

$$\Delta t = \gamma \Delta t_0,$$

where γ is the **relativistic factor** given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

and v and c are the speeds of the moving observer and light, respectively.

TIPS FOR SUCCESS

Try putting some values for v into the expression for the relativistic factor (γ). Observe at which speeds this factor will make a difference and when γ is so close to 1 that it can be ignored. Try 225 m/s, the speed of an airliner; 2.98×10^4 m/s, the speed of Earth in its orbit; and 2.990×10^8 m/s, the speed of a particle in an accelerator.

Notice that when the velocity v is small compared to the speed of light c, then v/c becomes small, and γ becomes close to 1. When this happens, time measurements are the same in both frames of reference. **Relativistic** effects, meaning those that have to do with special relativity, usually become significant when speeds become comparable to the speed of light. This is seen to be the case for time dilation.

You may have seen science fiction movies in which space travelers return to Earth after a long trip to find that the planet and everyone on it has aged much more than they have. This type of scenario is a based on a thought experiment, known as the twin paradox, which imagines a pair of twins, one of whom goes on a trip into space while the other stays home. When the space traveler returns, she finds her twin has aged much more than she. This happens because the traveling twin has been in two frames of reference, one leaving Earth and one returning.

Time dilation has been confirmed by comparing the time recorded by an atomic clock sent into orbit to the time recorded by a clock that remained on Earth. GPS satellites must also be adjusted to compensate for time dilation in order to give accurate positioning.

Have you ever driven on a road, like that shown in Figure 10.6, that seems like it goes on forever? If you look ahead, you might say you have about 10 km left to go. Another traveler might say the road ahead looks like it is about 15 km long. If you both measured the road, however, you would agree. Traveling at everyday speeds, the distance you both measure would be the same. You will read in this section, however, that this is not true at relativistic speeds. Close to the speed of light, distances measured are not the same when measured by different observers moving with respect to one other.

Figure 10.6 People might describe distances differently, but at relativistic speeds, the distances really are different. (Corey Leopold, Flickr)

One thing all observers agree upon is their relative speed. When one observer is traveling away from another, they both see the other receding at the same speed, regardless of whose frame of reference is chosen. Remember that speed equals distance divided by time: $v = d/t$. If the observers experience a difference in elapsed time, they must also observe a difference in distance traversed. This is because the ratio d/t must be the same for both observers.

The shortening of distance experienced by an observer moving with respect to the points whose distance apart is measured is called **length contraction**. **Proper length**, L_0, is the distance between two points measured in the reference frame where the observer and the points are at rest. The observer in motion with respect to the points measures L. These two lengths are related by the equation

$$L = \frac{L_0}{\gamma}.$$

Because γ is the same expression used in the time dilation equation above, the equation becomes

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}.$$

To see how length contraction is seen by a moving observer, go to this simulation (http://openstax.org/l/28simultaneity) . Here you can also see that simultaneity, time dilation, and length contraction are interrelated phenomena.

This link is to a simulation that illustrates the relativity of simultaneous events.

In classical physics, momentum is a simple product of mass and velocity. When special relativity is taken into account, objects that have mass have a speed limit. What effect do you think mass and velocity have on the momentum of objects moving at relativistic speeds; i.e., speeds close to the speed of light?

Momentum is one of the most important concepts in physics. The broadest form of Newton's second law is stated in terms of momentum. Momentum is conserved in classical mechanics whenever the net external force on a system is zero. This makes momentum conservation a fundamental tool for analyzing collisions. We will see that momentum has the same importance in modern physics. **Relativistic momentum** is conserved, and much of what we know about subatomic structure comes from the analysis of collisions of accelerator-produced relativistic particles.

One of the postulates of special relativity states that the laws of physics are the same in all inertial frames. Does the law of conservation of momentum survive this requirement at high velocities? The answer is yes, provided that the momentum is defined as follows.

Relativistic momentum, $\mathbf{p}$, is classical momentum multiplied by the relativistic factor γ.

$$\mathbf{p} = \gamma m\mathbf{u}, \tag{10.3}$$

where m is the **rest mass** of the object (that is, the mass measured at rest, without any γ factor involved), $\mathbf{u}$ is its velocity relative to an observer, and γ, as before, is the relativistic factor. We use the mass of the object as measured at rest because we cannot determine its mass while it is moving.

Note that we use $\mathbf{u}$ for velocity here to distinguish it from relative velocity $\mathbf{v}$ between observers. Only one observer is being considered here. With $\mathbf{p}$ defined in this way, $\mathbf{p}_{\text{tot}}$ is conserved whenever the net external force is zero, just as in classical physics. Again we see that the relativistic quantity becomes virtually the same as the classical at low velocities. That is, relativistic momentum $\gamma m\mathbf{u}$ becomes the classical $m\mathbf{u}$ at low velocities, because γ is very nearly equal to 1 at low velocities.

Relativistic momentum has the same intuitive feel as classical momentum. It is greatest for large masses moving at high velocities. Because of the factor γ, however, relativistic momentum behaves differently from classical momentum by approaching infinity as $\mathbf{u}$ approaches c. (See Figure 10.7.) This is another indication that an object with mass cannot reach the speed of light. If it did, its momentum would become infinite, which is an unreasonable value.

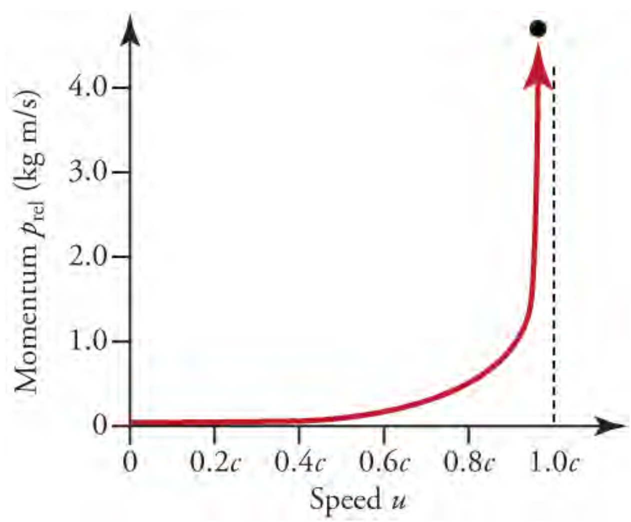

Figure 10.7 Relativistic momentum approaches infinity as the velocity of an object approaches the speed of light.

Relativistic momentum is defined in such a way that the conservation of momentum will hold in all inertial frames. Whenever the net external force on a system is zero, relativistic momentum is conserved, just as is the case for classical momentum. This has been verified in numerous experiments.

Mass-Energy Equivalence

Let us summarize the calculation of relativistic effects on objects moving at speeds near the speed of light. In each case we will need to calculate the relativistic factor, given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}},$$

where **v** and c are as defined earlier. We use **u** as the velocity of a particle or an object in one frame of reference, and **v** for the velocity of one frame of reference with respect to another.

Time Dilation

Elapsed time on a moving object, Δt_0, as seen by a stationary observer is given by $\Delta t = \gamma \Delta t_0$, where Δt_0 is the time observed on the moving object when it is taken to be the frame or reference.

Length Contraction

Length measured by a person at rest with respect to a moving object, L, is given by

$$L = \frac{L_0}{\gamma},$$

where L_0 is the length measured on the moving object.

Relativistic Momentum

Momentum, $\mathbf{p}$, of an object of mass, m, traveling at relativistic speeds is given by $\mathbf{p} = \gamma m \mathbf{u}$, where **u** is velocity of a moving object as seen by a stationary observer.

Relativistic Energy

The original source of all the energy we use is the conversion of mass into energy. Most of this energy is generated by nuclear reactions in the sun and radiated to Earth in the form of electromagnetic radiation, where it is then transformed into all the forms with which we are familiar. The remaining energy from nuclear reactions is produced in nuclear power plants and in Earth's interior. In each of these cases, the source of the energy is the conversion of a small amount of mass into a large amount of energy. These sources are shown in Figure 10.8.

Figure 10.8 The sun (a) and the Susquehanna Steam Electric Station (b) both convert mass into energy. ((a) NASA/Goddard Space Flight Center, Scientific Visualization Studio; (b) U.S. government)

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Einstein showed that the law of conservation of energy is valid relativistically, if we define energy to include a relativistic factor. The result of his analysis is that a particle or object of mass m moving at velocity **u** has **relativistic energy** given by

$$E = \gamma mc^2.$$

This is the expression for the total energy of an object of mass m at any speed **u** and includes both kinetic and potential energy. Look back at the equation for γ and you will see that it is equal to 1 when **u** is 0; that is, when an object is at rest. Then the rest energy, E_0, is simply

$$E_0 = mc^2.$$

This is the correct form of Einstein's famous equation.

This equation is very useful to nuclear physicists because it can be used to calculate the energy released by a nuclear reaction. This is done simply by subtracting the mass of the products of such a reaction from the mass of the reactants. The difference is the m in $E_0 = mc^2$. Here is a simple example:

A positron is a type of antimatter that is just like an electron, except that it has a positive charge. When a positron and an electron collide, their masses are completely annihilated and converted to energy in the form of gamma rays. Because both particles have a rest mass of 9.11×10^{-31} kg, we multiply the mc^2 term by 2. So the energy of the gamma rays is

$$\begin{aligned} E_0 &= 2(9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^8\ \tfrac{\text{m}}{\text{s}}\right)^2 \\ &= 1.64 \times 10^{-13}\ \tfrac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \\ &= 1.64 \times 10^{-13}\ \text{J} \end{aligned} \tag{10.4}$$

where we have the expression for the joule (J) in terms of its SI base units of kg, m, and s. In general, the nuclei of stable isotopes have less mass then their constituent subatomic particles. The energy equivalent of this difference is called the **binding energy** of the nucleus. This energy is released during the formation of the isotope from its constituent particles because the product is more stable than the reactants. Expressed as mass, it is called the **mass defect**. For example, a helium nucleus is made of two neutrons and two protons and has a mass of 4.0003 atomic mass units (u). The sum of the masses of two protons and two neutrons is 4.0330 u. The mass defect then is 0.0327 u. Converted to kg, the mass defect is 5.0442×10^{-30} kg. Multiplying this mass times c^2 gives a binding energy of 4.540×10^{-12} J. This does not sound like much because it is only one atom. If you were to make one gram of helium out of neutrons and protons, it would release 683,000,000,000 J. By comparison, burning one gram of coal releases about 24 J.

BOUNDLESS PHYSICS

The RHIC Collider

Figure 10.9 shows the Brookhaven National Laboratory in Upton, NY. The circular structure houses a particle accelerator called the RHIC, which stands for Relativistic Heavy Ion Collider. The heavy ions in the name are gold nuclei that have been stripped of their electrons. Streams of ions are accelerated in several stages before entering the big ring seen in the figure. Here, they are accelerated to their final speed, which is about 99.7 percent the speed of light. Such high speeds are called relativistic. All the relativistic phenomena we have been discussing in this chapter are very pronounced in this case. At this speed $\gamma = 12.9$, so that relativistic time dilates by a factor of about 13, and relativistic length contracts by the same factor.

Figure 10.9 Brookhaven National Laboratory. The circular structure houses the RHIC. (energy.gov, Wikimedia Commons)

Two ion beams circle the 2.4-mile long track around the big ring in opposite directions. The paths can then be made to cross, thereby causing ions to collide. The collision event is very short-lived but amazingly intense. The temperatures and pressures produced are greater than those in the hottest suns. At 4 trillion degrees Celsius, this is the hottest material ever created in a laboratory

But what is the point of creating such an extreme event? Under these conditions, the neutrons and protons that make up the gold nuclei are smashed apart into their components, which are called quarks and gluons. The goal is to recreate the conditions that theorists believe existed at the very beginning of the universe. It is thought that, at that time, matter was a sort of soup of quarks and gluons. When things cooled down after the initial bang, these particles condensed to form protons and neutrons.

Some of the results have been surprising and unexpected. It was thought the quark-gluon soup would resemble a gas or plasma. Instead, it behaves more like a liquid. It has been called a *perfect* liquid because it has virtually no viscosity, meaning that it has no resistance to flow.

GRASP CHECK

Calculate the relativistic factor γ, for a particle traveling at 99.7 percent of the speed of light.

a. 0.08
b. 0.71
c. 1.41
d. 12.9

WORKED EXAMPLE

The Speed of Light

One night you are out looking up at the stars and an extraterrestrial spaceship flashes across the sky. The ship is 50 meters long and is travelling at 95 percent of the speed of light. What would the ship's length be when measured from your earthbound frame of reference?

Strategy

List the knowns and unknowns.

Knowns: proper length of the ship, $L_0 = 50$ m; velocity, $\mathbf{v}$, $= 0.95c$

Unknowns: observed length of the ship accounting for relativistic length contraction, L.

Choose the relevant equation.

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1 - \frac{\mathbf{u}^2}{c^2}}$$

Solution

$$L = 50\text{ m}\sqrt{1 - \frac{(0.95)^2 c^2}{c^2}} = 50\text{ m}\sqrt{1 - (0.95)^2} = 16\text{ m}$$

Discussion

Calculations of γ can usually be simplified in this way when v is expressed as a percentage of c because the c^2 terms cancel. Be sure to also square the decimal representing the percentage before subtracting from 1. Note that the aliens will still see the length as L_0 because they are moving with the frame of reference that is the ship.

Practice Problems

7. Calculate the relativistic factor, γ, for an object traveling at 2.00×10^8 m/s.
   a. 0.74
   b. 0.83
   c. 1.2
   d. 1.34

8. The distance between two points, called the proper length, L0, is 1.00 km. An observer in motion with respect to the frame of

reference of the two points measures 0.800 km, which is L. What is the relative speed of the frame of reference with respect to the observer?

a. 1.80×10^8 m/s
b. 2.34×10^8 m/s
c. 3.84×10^8 m/s
d. 5.00×10^8 m/s

9. Consider the nuclear fission reaction $n + {}^{235}_{92}U \rightarrow {}^{137}_{55}Cs + {}^{97}_{37}Rb + 2n + E$. If a neutron has a rest mass of 1.009u, ${}^{235}_{92}U$ has a rest mass of 235.044u, ${}^{137}_{55}Cs$ has rest mass of 136.907u, and ${}^{97}_{37}Rb$ has a rest mass of 96.937u, what is the value of E in joules?

a. 1.8×10^{-11} J
b. 2.9×10^{-11} J
c. 1.8×10^{-10} J
d. 2.9×10^{-10} J

Solution

The correct answer is (b). The mass deficit in the reaction is $235.044\ \text{u} - (136.907 + 96.937 + 1.009)\ \text{u}$, or 0.191u. Converting that mass to kg and applying $E = mc^2$ to find the energy equivalent of the mass deficit gives

$(0.191\ \text{u})\left(1.66 \times 10^{-27}\text{kg/u}\right)\left(3.00 \times 10^8\text{m/s}\right)^2 \cong 2.85 \times 10^{-11}\text{J}.$

10. Consider the nuclear fusion reaction ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}H + E$. If ${}^{2}_{1}H$ has a rest mass of 2.014u, ${}^{3}_{1}H$ has a rest mass of 3.016u, and ${}^{1}_{1}H$ has a rest mass of 1.008u, what is the value of E in joules?

a. 6×10^{-13} J
b. 6×10^{-12} J
c. 6×10^{-11} J
d. 6×10^{-10} J

Solution

The correct answer is (a). The mass deficit in the reaction is $2\,(2.014\ \text{u}) - (3.016 + 1.008)\ \text{u}$, or 0.004u. Converting that mass to kg and applying $E = mc^2$ to find the energy equivalent of the mass deficit gives

$(0.004\ \text{u})\left(1.66 \times 10^{-27}\text{kg/u}\right)\left(3.00 \times 10^8\text{m/s}\right)^2 \cong 5.98 \times 10^{-13}\text{J}.$

Check Your Understanding

11. Describe time dilation and state under what conditions it becomes significant.

a. When the speed of one frame of reference past another reaches the speed of light, a time interval between two events at the same location in one frame appears longer when measured from the second frame.
b. When the speed of one frame of reference past another becomes comparable to the speed of light, a time interval between two events at the same location in one frame appears longer when measured from the second frame.
c. When the speed of one frame of reference past another reaches the speed of light, a time interval between two events at the same location in one frame appears shorter when measured from the second frame.
d. When the speed of one frame of reference past another becomes comparable to the speed of light, a time interval between two events at the same location in one frame appears shorter when measured from the second frame.

12. The equation used to calculate relativistic momentum is $p = \gamma \cdot m \cdot u$. Define the terms to the right of the equal sign and state how m and u are measured.

a. γ is the relativistic factor, m is the rest mass measured when the object is at rest in the frame of reference, and u is the velocity of the frame.
b. γ is the relativistic factor, m is the rest mass measured when the object is at rest in the frame of reference, and u is the velocity relative to an observer.
c. γ is the relativistic factor, m is the relativistic mass $\left(\text{i.e.,} \frac{m}{\sqrt{1-\frac{u^2}{c^2}}}\right)$ measured when the object is moving in the frame of reference, and u is the velocity of the frame.

   d. γ is the relativistic factor, m is the relativistic mass $\left(\text{i.e.,} \frac{m}{\sqrt{1-\frac{u^2}{c^2}}}\right)$ measured when the object is moving in the frame of reference, and u is the velocity relative to an observer.

13. Describe length contraction and state when it occurs.
   a. When the speed of an object becomes the speed of light, its length appears to shorten when viewed by a stationary observer.
   b. When the speed of an object approaches the speed of light, its length appears to shorten when viewed by a stationary observer.
   c. When the speed of an object becomes the speed of light, its length appears to increase when viewed by a stationary observer.
   d. When the speed of an object approaches the speed of light, its length appears to increase when viewed by a stationary observer.